A Simple Quasi-Static Determination of Basic Parameters of Multilayer Microstrip and Coplanar Waveguide

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***Abstract*— Two generalized multilayer transmission lines are investigated: microstrip line (MSL) and asymmetrical coplanar waveguide (CPW). The conformal mapping method is used to obtain simple analytical expressions for quasi-TEM effective permittivity of these structures. Accuracy of derived generalized formulas is verified for any special cases of multilayer structures.**

I. Introduction

FOR relative simple but sufficiently accurate investigation of conventional (i.e., one-dielectric layer) microstrip and coplanar lines, the conformal mapping methods are successfully used [1], [2]. In the contributions [3], [4] we have shown any applications of this techniques to solving various types of three-layer microstrips and coplanar lines. In this letter, the relations are presented for calculating the basic quasi-static parameters of generalized multilayer MSL and CPW's.

II. Multilayer Microstrip

This type of N homogenous and isotropic dielectric layers microstrip will be mapped from Fig. 1(a) onto another complex variable plane with result as shown in Fig. 1(b) [3]. For the filling factors of the individual dielectric layers we obtain: for a wide strip $W/h \geq 1$

$$q_1 = \frac{H_1}{2} \cdot \left[1 + \frac{\pi}{4} - \frac{h}{W_{ef}} \cdot \ln\left(2\frac{W_{ef}}{h} \cdot \frac{\sin\left(\frac{\pi}{2} \cdot H_1\right)}{H_1} + \cos\left(\frac{\pi}{2} \cdot H_1\right)\right)\right], \tag{1}$$

$$q_i = (1), \quad \text{for } H_i - (1), \quad \text{for } H_{i-1}, \quad i = 2, 3, \cdots, M-1 \tag{2}$$

$$q_M = 1 - \frac{h}{2W_{ef}} \cdot \ln\left(\pi\frac{W_{ef}}{h} - 1\right) - (1), \quad \text{for } H_{M-1}, \tag{3}$$

$$q_{M+1} = \frac{h}{2W_{ef}} \cdot \left\{\ln\left(\pi \cdot \frac{W_{ef}}{h} - 1\right) - (1 + V_{M+1}) \times \ln\left[2\frac{W_{ef}}{h} \cdot \frac{\cos\left(\frac{\pi}{2} \cdot V_{M+1}\right)}{2H_{M+1} - 1 + V_{M+1}} + \sin\left(\frac{\pi}{2} \cdot V_{M+1}\right)\right]\right\}, \tag{4}$$

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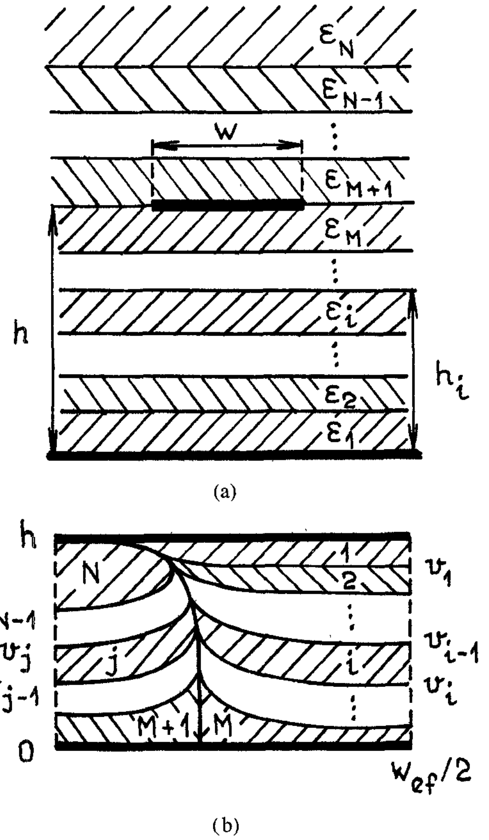

Fig. 1. Multilayer microstrip. (a) Initial structure. (b) Conformal mapped structure.

$$q_j = (4), \quad \text{for } (H_j, V_j) - (4), \quad \text{for } (H_{j-1}, V_{j-1}), \quad j = M+2, M+3, \cdots, N-1 \tag{5}$$

$$q_N = 1 - \sum_{j=M+1}^{N-1} q_j - \sum_{i=1}^{M} q_i \tag{6}$$

In these relations, denote $H_{i,j} = h_{i,j}/h$ and $V_j = v_j/h$, where from Fig. 1(b)

$$v_j = \frac{2h}{\pi} \cdot \arctan\left[\frac{\pi}{\frac{\pi}{2} \cdot \frac{W_{ef}}{h} - 2}(H_j - 1)\right] \tag{7}$$

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and the effective line width is

$$W_{ef} = W + \frac{2h}{\pi} \cdot \ln\left[17.08\left(\frac{W}{2h} + 0.92\right)\right]. \quad (8)$$

For a narrow microstrip line, $W/h \leq 1$, we have

$$q_1 = \frac{\ln A_1}{2 \cdot \ln \frac{8h}{W}} \cdot \left[1 + \frac{\pi}{4} - \frac{1}{2} \cdot \arccos\left(\frac{W}{8hH_1} \cdot \sqrt{A_1}\right)\right], \quad (9)$$

$$q_i = (9), \quad \text{for } (H_i, A_i) - (9), \quad \text{for } (H_{i-1}, A_{i-1}),$$
$$i = 2, 3, \cdots, M-1, \quad (10)$$

$$q_M = \frac{1}{2} + \frac{0.9}{\pi \cdot \ln \frac{8h}{W}} - (9), \quad \text{for } (H_{M-1}, A_{M-1}), \quad (11)$$

$$q_{M+1} = \frac{1}{2} - \frac{0.9 + \frac{\pi}{4} \cdot \ln B_{M+1} \cdot \arccos\left[\left(1 - \frac{1-\frac{W}{8h}}{H_{M+1}}\right) \cdot \sqrt{B_{M+1}}\right]}{\pi \cdot \ln \frac{8h}{W}}, \quad (12)$$

$$q_j = (12), \quad \text{for } (H_j, B_j) - (12), \quad \text{for } (H_{j-1}, B_{j-1}),$$
$$j = M+2, M+3, \cdots, N-1, \quad (13)$$

and q_N is given by (6). Further, denote

$$A_i = \frac{1 + H_i}{1 - H_i + \frac{W}{4h}}, \qquad B_j = \frac{H_j + 1}{H_j + \frac{W}{4h} - 1}. \quad (14)$$

From Fig. 1(b), we derive a general relation for effective permittivity of multilayer microstrip

$$\varepsilon_{efr} = \frac{\left(\sum_{i=1}^{M} q_i\right)^2}{\sum_{i=1}^{M} \frac{q_i}{\varepsilon_{ri}}} + \frac{\left(\sum_{j=M+1}^{N} q_j\right)^2}{\sum_{j=M+1}^{N} \frac{q_j}{\varepsilon_{rj}}}. \quad (15)$$

This equation include all special cases of multilayer MSL's, such as a conventional microstrip on one-layer dielectric substrate ($M = 1, N = 2, \varepsilon_{r2} = 1$), a inverted microstrip ($M = 1, N = 3, \varepsilon_{r1} = \varepsilon_{r3} = 1$), and a suspended microstrip ($M = 2, N = 3, \varepsilon_{r1} = \varepsilon_{r3} = 1$). For these special cases, the accuracy of the previous relations was verified [3] through comparison our results with calculated and measured values from [5], [6]. The agreement was found to be very good.

III. Multilayer Asymmetric Coplanar Waveguide

Here, we transform the conducting strips and all individual dielectric layers from Fig. 2(a) to a approximate equivalent formation in Fig. 2(b). That dimensions are given as

$$\frac{v_0}{u_o} = 2 \cdot \frac{K'(k)}{K(k)}, \qquad \frac{v_0}{u_i} = 2 \cdot \frac{K'(k_i)}{K(k_i)}, \quad (16)$$

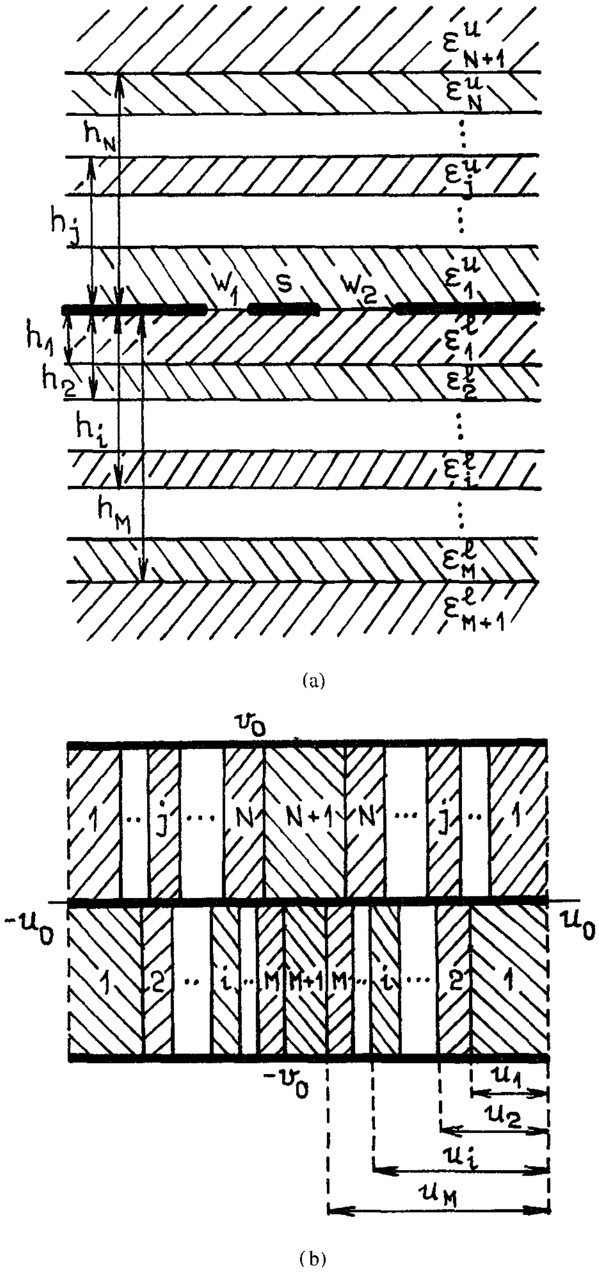

Fig. 2. Multilayer asymmetric coplanar waveguide. (a) Initial structure. (b) Approximately conformal mapped structure.

where K and K' are complete elliptic integrals of the first kind with modulus

$$k_i^2 = 2 \cdot \frac{k_{i1} + k_{i2}}{(1 + k_{i1}) \cdot (1 + k_{i2})}, \quad (16a)$$

$$k_{i1} = \frac{\sinh\left(\frac{\pi}{4} \cdot \frac{s}{h_i}\right)}{\sinh\left[\frac{\pi}{4h_i}(s + 2W_1)\right]}, \qquad k_{i2} = \frac{\sinh\left(\frac{\pi}{4} \cdot \frac{s}{h_i}\right)}{\sinh\left[\frac{\pi}{4h_i}(s + 2W_2)\right]} \quad (16b)$$

and $k = k_{M+1} = k_{N+1}$ (for $h_{M+1}, h_{N+1} \to \infty$). From Fig. 2(b), the individual filling factors are

$$q_i = \frac{K'(k)}{2 \cdot K(k)} \cdot \left[\frac{K(k_i)}{K'(k_i)} - \frac{K(k_{i-1})}{K'(k_{i-1})}\right], \quad (17)$$

for $k_0 = 0$ and $i = 1, 2, \cdots, M+1$ and $i = 1, 2, \cdots, N+1$, respectively. For the relative effective permittivity of the multilayer asymmetric CPW, we have

$$\varepsilon_{efr} = \sum_{i=1}^{M+1} \varepsilon_{ri}^{\ell} \cdot q_i + \sum_{j=1}^{N+1} \varepsilon_{rj}^{u} \cdot q_j. \tag{18}$$

From (17) and (18), for $M = 1$, $N = 0$, $\varepsilon_{r2}^{\ell} = \varepsilon_{r1}^{u} = 1$ and $W_1 = W_2$, we obtain the well-known relation [2] for symmetrical CPW on one-layer dielectric substrate. Further special CPW's that basic parameters can be derived from (17) and (18) are: a sandwich asymmetric CPW ($M = N = 1$, $\varepsilon_{r2}^{\ell} = \varepsilon_{r2}^{u} = 1$) and a supported CPW on a double-layers composite substrate ($M = 2$, $N = 0$, $\varepsilon_{r3}^{\ell} = \varepsilon_{r1}^{u} = 1$), which is often used in millimeter waves region. For a symmetrical ($W_1 = W_2$), three-layer CPW (17) and (18) are identical with formulas given in [7].

IV. Conclusion

The conformal mapping method provide good and simple analytical results in both investigated multilayer MIC's. While in cases of more than three dielectric layers the complexity and computation time of exact numerical calculations grows very rapidly, our calculation does not become more complicated because the equations used are permanent the same.

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